Eur. Phys. J. B **48**, 101–106 (2005)
DOI: 10.1140/epjb/e2005-00383-0

THE EUROPEAN PHYSICAL JOURNAL B

Heat flow and efficiency in a microscopic engine

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Received 6 September 2005
Published online 9 December 2005 –

Abstract. We study the energetics of a thermal motor driven by temperature differences, which consists of a Brownian particle moving in a sawtooth potential with an external load where the viscous medium is periodically in contact with hot and cold heat reservoir along space coordinate. The motor can work as a heat engine or a refrigerator under different conditions. The heat flow via both potential and kinetic energy is considered. The former is reversible when the engine works quasistatically and the latter is always irreversible. The efficiency of the heat engine can never approach Carnot efficiency.

PACS. 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 05.70.-a Thermodynamics – 87.10.+e General theory and mathematical aspects

1 Introduction

Recently, Brownian ratchets (motors) have attracted considered attention simulated by research on molecular motors [1,2]. A Brownian ratchet , which appeared in Feynman's famous textbook for the first time as a thermal ratchet [3], is a machine which can rectify thermal fluctuation and produce a directed current. These subjects are motivated by the challenge to understand molecular motors [4], nanoscale friction [5], surface smoothening [6], coupled Josephson junctions [7], optical ratchets and directed motion of laser cooled atoms [8], and mass separation and trapping schemes at the microscale [9].

Brownian ratchets are spatially asymmetric but periodic structure in which transport of Brownian particles is induced by some nonequilibrium processes [10–15]. Typical examples are external modulation of underling potential, a nonequilibrium chemical reaction coupled to a change of the potential and contact with reservoirs at different temperatures. The most studies have been on the velocity of the transport particle. Another important aspect is efficiency of energy conversion.

Recently, Sekimoto has been unambiguously defined the concept of the heat at mesoscopic scales in terms of Langevin equation [16]. He refers to the formalism providing this definition as stochastic energetics. The essential point of this formalism is that the heat transferred to the system is nothing but the microscopic work done by both friction force and the random force in the Langevin equation. Stochastic energetics has been applied to the study of thermodynamic processes. Derenyi and Astumian [17] have studied the efficiency of one-dimensional thermally driven Brownian engines. They identify and compare the three basic setups characterized by the type of the connection between the Brownian particle and the two reservoirs: (1) simultaneous [3]; (2) alternating in time [18]; and (3) position dependent [19]. Parrondo and Cisneros [15] has reviewed the literature the energetics of Brownian motors, distinguishing between forced ratchet, chemical motors-driven out of equilibrium by difference of chemical potential, and thermal motors-driven by temperature differences. In this paper we give a detailed study of the last motors-thermal motors.

Energetics of the thermal motors-driven by temperature differences are investigated by some previous works [20,21]. Asfaw et al. [20] have explored the performance of the motors under various conditions of practical interest such as maximum power and optimized efficiency. They found that the efficiency can approach the Carnot efficiency under the quasistatic limit. The same results are also obtained in Matsuo and Sasa's work [21] by stochastic energetics theory. The previous works are limited to case of heat flow via potential. The present work extends the study to case of heat flow via both potential and kinetics energy. The efficiency of heat engine is different from the results of previous works and can never approach the Carnot efficiency. The heat flow via potential is reversible when the engine works at quasistatic limit. The heat flow via the kinetic is always irreversible in nature.

2 Thermal motor works as a heat engine

Consider a Brownian particle moving in a sawtooth potential with a external load force F. The medium is periodically in contact with two heat reservoirs along the

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space coordinate (shown in Fig. 1). There are two different driving factors in the motor. The first one is noise-induced transport, namely, ratchet effect. The second one is that temperature difference makes the particle move from high temperature reservoir to low temperature reservoir.

Let E be the barrier height of the potential. L_1, L_2 are the length of the left part and the right part of the ratchet, respectively. $E + fL_1$ is the energy needed for a forward jump, while $E - fL_2$ is the energy required for a backward jump. The left part of a period ratchet is at temperature T_H (hot reservoir) and the right one is at temperature T_C (cold reservoir). We can assume that the rates of both forward and backward jumps are proportional to the corresponding Arrhenius' factor [22], such that

$$\dot{N}_+ = \frac{1}{t} \exp\left[-\frac{E + fL_1}{k_B T_H}\right], \tag{1}$$

$$\dot{N}_- = \frac{1}{t} \exp\left[-\frac{E - fL_2}{k_B T_C}\right], \tag{2}$$

are the number of forward and backward jumps per unit time, respectively, k_B the Boltzmann constant, t a proportionality constant with dimensions of time.

If $\dot{N}_+ > \dot{N}_-$, the ratchet works as a two-reservoir heat engine shown in Figure 2. The rate of heat flow from hot reservoir to the heat engine via potential is given by

$$\dot{Q}_H^{pot} = (\dot{N}_+ - \dot{N}_-)(E + fL_1). \tag{3}$$

The rate of heat flow from the engine to the cold reservoir via potential is

$$\dot{Q}_C^{pot} = (\dot{N}_+ - \dot{N}_-)(E - fL_2). \tag{4}$$

The heat flow via the kinetic energy of the particle is much more complicated to determined [17]. Whenever a particle stay at a hot segment (temperature T_H) it absorbs $\frac{1}{2}k_B T_H$ energy on average from the hot reservoir. It can pick up $\frac{1}{2}k_B T_C$ energy on average from the cold reservoir when the particle stay at a cold segment. It is obvious that when a particle leaves from a hot segment to a cold segment and then returns to the hot segment, the hot reservoir will lost $\frac{1}{2}(k_B T_H - k_B T_C)$ energy on average, the lost energy is released to the colder reservoir and never to the hot reservoir or to the particle's potential energy, which indicates the inherently irreversible nature of this heat flow. On the other hand, if a particle goes out from a hot segment to a cold segment and never returns to the hot segment, the hot reservoir will lost $\frac{1}{2}k_B T_H$ energy on average. In a unit time, $\dot{N}_+$ particles will absorb $\frac{1}{2}\dot{N}_+ k_B T_H$ energy from the hot reservoir and $\dot{N}_-$ particles will absorb $\frac{1}{2}\dot{N}_- k_B T_C$ energy from the cold reservoir. Therefore the rate of net heat flow via kinetic energy from the hot reservoir to the cold reservoir is given by

$$\dot{Q}^{kin} = \frac{1}{2}\dot{N}_+ k_B T_H - \frac{1}{2}\dot{N}_- k_B T_C. \tag{5}$$

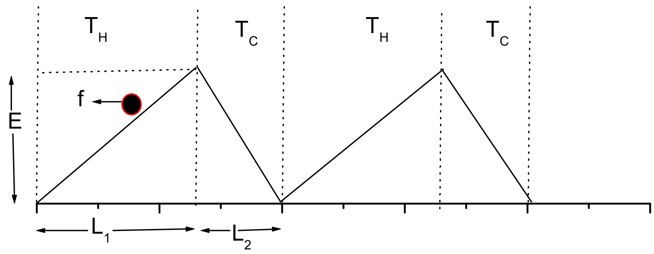


Fig. 1. Schematic illustration of the motor: a Brownian particle moves in a sawtooth potential with an external load where the medium is periodically in contact with two reservoirs along the space coordinate. The period of the potential is $L = L_1 + L_2$. The temperature profiles are also shown.

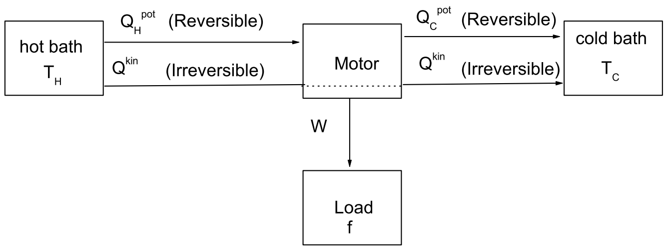


Fig. 2. The engine acts as a heat engine. Heat flows via both potential energy and kinetic energy in a thermal motor in contact with two thermal baths at temperatures $T_H > T_C$, W is power output, heat flows via potential energy is reversible, heat flows via kinetic energy is irreversible.

Therefore, the rate of total heat transferred from the hot reservoir is given by

$$\dot{Q}_H = \dot{Q}_H^{plot} + \dot{Q}^{kin} = (\dot{N}_+ - \dot{N}_-) \times (E + fL_1) + \frac{1}{2}\dot{N}_+ k_B T_H - \frac{1}{2}\dot{N}_- k_B T_C. \tag{6}$$

The rate of total heat transferred to the cold reservoir is

$$\dot{Q}_C = \dot{Q}_C^{plot} + \dot{Q}^{kin} = (\dot{N}_+ - \dot{N}_-) \times (E - fL_2) + \frac{1}{2}\dot{N}_+ k_B T_H - \frac{1}{2}\dot{N}_- k_B T_C. \tag{7}$$

The power output is

$$\dot{W} = \dot{Q}_H - \dot{Q}_C = (\dot{N}_+ - \dot{N}_-) fL. \tag{8}$$

It is easy to obtain the efficiency of the heat engine

$$\eta = \frac{\dot{W}}{\dot{Q}_H}. \tag{9}$$

If the heat flow via the kinetic energy is not considered, the efficiency is given by

$$\eta^{pot} = \frac{\dot{W}}{\dot{Q}_H^{pot}}. \tag{10}$$

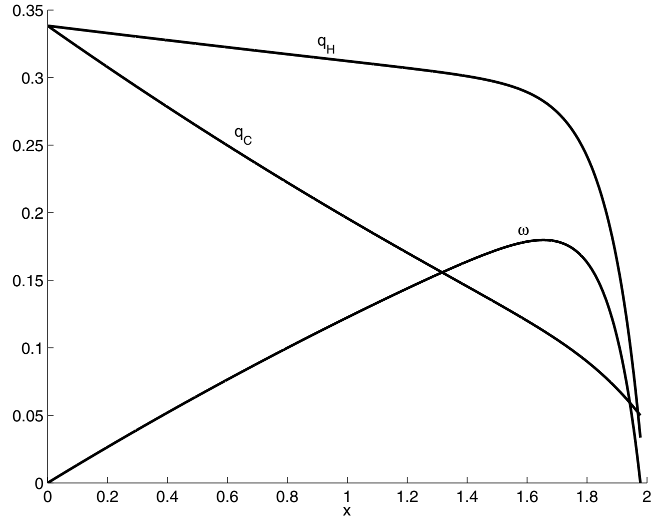


Fig. 3. Dimensionless heat flow q_H,q_C and power output w vs. the load x ($\tau = 0.1$, $\epsilon = 2.0$, $\mu = 0.1$).

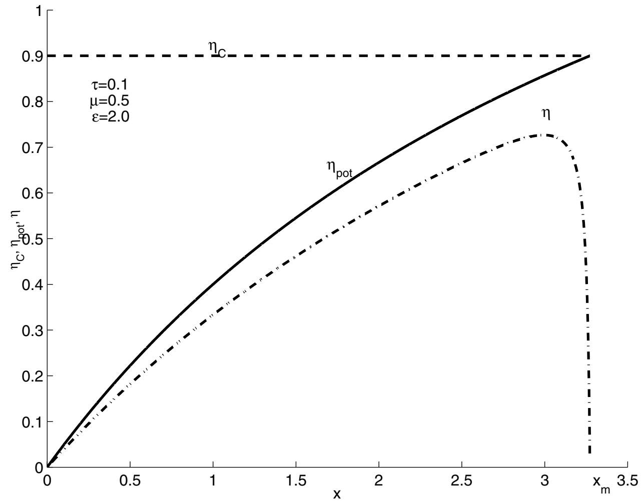


Fig. 4. η_C, η_{pot} and η vs. the load x ($\tau = 0.1$, $\epsilon = 2.0$, $\mu = 0.1$).

In order to discuss simply, we can rewritten equations (5–10) in a dimensionless form, then we get

$$q_H = \frac{\dot{Q}_H t}{k_B T_H} = e^{-\epsilon/\tau}\left[\left(\epsilon + \frac{1}{2} + \mu x\right)e^{x_0 - \mu x} - \left(\epsilon + \frac{1}{2}\tau + \mu x\right)e^{(1-\mu)x/\tau}\right] \quad (11)$$

$$q_C = e^{-\epsilon/\tau}\left[\left(\epsilon + \frac{1}{2} + \mu x - x\right)e^{x_0 - \mu x} - \left(\epsilon + \frac{1}{2}\tau + \mu x - x\right)e^{(1-\mu)x/\tau}\right] \quad (12)$$

$$q^{kin} = \frac{\dot{Q}_{kin} t}{k_B T_H} = \frac{1}{2}e^{-\epsilon/\tau}[e^{x_0 - \mu x} - \tau e^{(1-\mu)x/\tau}] \quad (13)$$

$$w = \frac{\dot{W} t}{k_B T_H} = e^{-\epsilon/\tau}[e^{x_0 - \mu x} - e^{(1-\mu)x/\tau}]x \quad (14)$$

$$\eta = \frac{[e^{x_0 - \mu x} - e^{(1-\mu)x/\tau}]x}{(\epsilon + \mu x + \frac{1}{2})e^{x_0 - \mu x} - (\epsilon + \mu x + \frac{1}{2}\tau)e^{(1-\mu)x/\tau}} \quad (15)$$

$$\eta^{pot} = \frac{x}{\epsilon + \mu x}, \quad (16)$$

where

$$x = \frac{fL}{k_B T_H}, \epsilon = \frac{E}{k_B T_H}, \tau = \frac{T_C}{T_H}, \mu = \frac{L_1}{L}, x_0 = \frac{(1-\tau)\epsilon}{\tau}. \quad (17)$$

From the above equations, one has $\epsilon \geq 0$, $0 \leq \tau \leq 1$, $0 \leq \mu \leq 1$. Since the motor acts as a heat engine, the power output can not be negative, namely, $w \geq 0$. one can also obtain the x value range of x, $0 \leq x \leq x_m$, where

$$x_m = \frac{(1-\tau)\epsilon}{(\tau - 1)\mu + 1}. \quad (18)$$

Therefore, it is easy to obtain

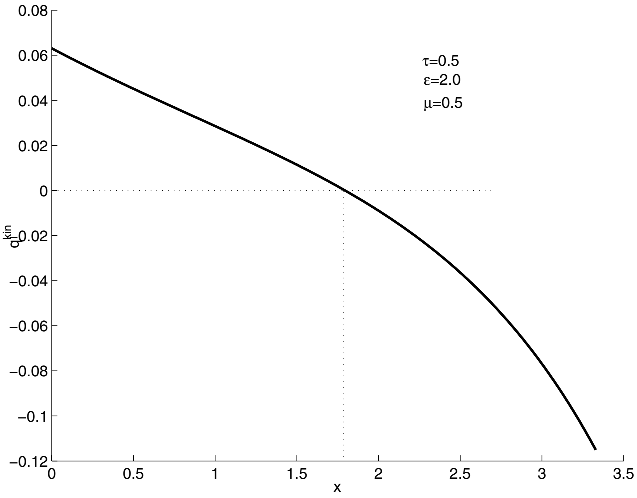
$$\eta^{pot} = \frac{x}{\epsilon + \mu x} \leq \frac{x_m}{\epsilon + \mu x_m} = 1 - \tau = 1 - \frac{T_C}{T_H} = \eta_C, \quad (19)$$

where η_C is Carnot efficiency. However, η^{pot} attains the Carnot efficiency, namely, $x = x_m$ which indicates that the power output is zero. From equations (15) and (16), it obvious that η is always less than η^{pot}. The results are given by Figures 3–10.

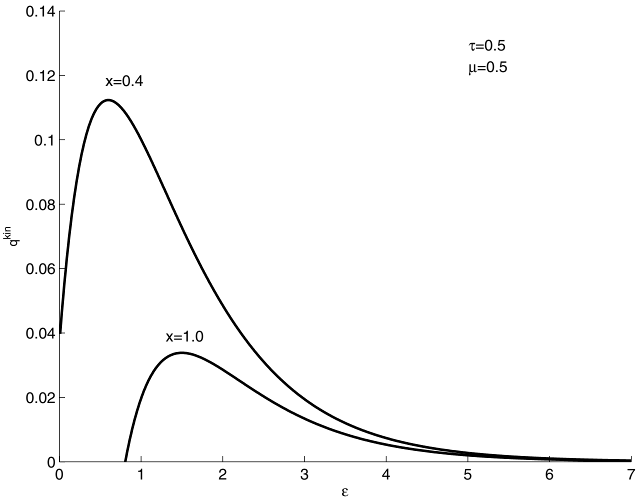
Figure 3 shows that the heat flow q_H, q_C and power output w as a function of the load x. When $x = 0$, namely, the engine runs without a load, q_H is equal to q_C, which indicates that the heat that absorbs from the hot reservoir releases to the cold reservoir entirely and no power output is obtained. When x increases, q_H and q_C decreases. when $x \to 0$, no power output is obtained (w=0). When $x \to x_m$, no currents occur, the ratchet can't give any power output. So the power output w has a maximum value at certain value of x which depends on τ, ϵ and μ.

Figure 4 shows the variation of the efficiency η_C, η_{pot}, η with the load x. If the heat flow via kinetic energy is ignored, the efficiency η_{pot} increases with the load x, it approaches the Carnot efficiency η_C at quasistatic condition ($x = x_m$). The result is also presented in Asfaw's work [20]. When the heat flow via kinetic energy are considered, the curve of the efficiency η is observed to be bell-shaped, a feature of resonance. The efficiency η is always less than η_{pot} and never approaches Carnot efficiency η_C. It is obvious that the heat flow via kinetic energy is always irreversible and energy leakage is inevitable, so the efficiency is less than η_{pot} and can't approach Carnot efficiency.

Figure 5a shows the heat flow q^{kin} out of hot reservoir via kinetic energy as a function of the load x. When $x < x_m$, q^{kin} is positive. When $x > x_m$, q^{kin} is negative. No heat flow occurs at $x = x_m$. It is found that the heat flow via kinetic is dependant on the current of the ratchet. Figure 5b shows the heat flow out of the hot reservoir via kinetic energy as a function of barrier height ϵ. The curve



(a)



(b)

Fig. 5. (a) Dimensionless heat flow q^{kin} vs. the load x ($\tau = 0.5$. $\epsilon = 2.0$, $\mu = 0.5$). (b) Dimensionless heat flow q^{kin} vs. barrier height ϵ for different values of $x = 0.4, 1.0$ ($\tau = 0.5$, $\mu = 0.5$).

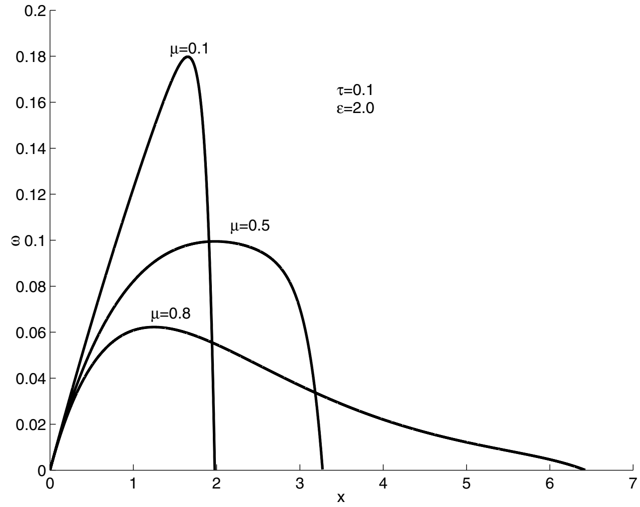


Fig. 6. Dimensionless power output w vs. the load x for different values of $\mu = 0.1, 0.5, 0.8$ ($\tau = 0.1$, $\epsilon = 2.0$).

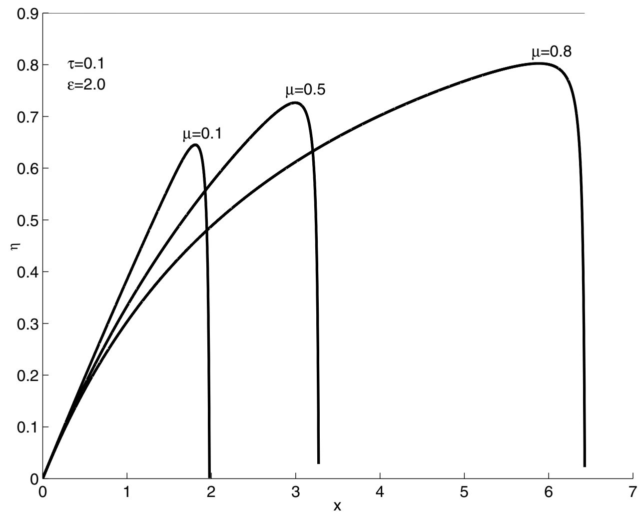


Fig. 7. Efficiency η vs. the load x for different values of $\mu = 0.1, 0.5, 0.8$ ($\tau = 0.1$, $\epsilon = 2.0$).

is bell-shaped. Therefore, there is an optimized value of ϵ at which q^{kin} takes its maximum value.

Figure 6 shows the power output w as a function of the load x for different values of $\mu = 0.1, 0.5, 0.8$. From the figure, we can see that the power output has a maximum value at certain value of x. The maximum load x_m of the engine changes with the parameter μ of asymmetry in potential. In other word, the structure of the potential determines the maximum load capability of the engine.

Figure 7 shows the variation of the efficiency η with the load x for different values of $\mu = 0.1, 0.5, 0.8$. The maximum value of η increases with μ. However, the maximum value of η can't approach the Carnot efficiency.

Figure 8 shows the power output w as a function of the barrier height ϵ for different values of the load $x = 0.1, 0.2, 0.5$. When $\epsilon \to 0$, the effect of ratchet disappears, the power output tends to zero. When $\epsilon \to +\infty$ so that the particle can't pass the barrier, the power output w goes to zero, too. The power output w has a maximum value at certain value of ϵ which is dependant on τ, μ and x. On the other hand, the minimum height of the barrier for working as a heat engine increases with the load x.

Figure 9 shows the efficiency η_C, η_{pot} and η as a function of the barrier height. The efficiency η_{pot} without the heat flow via kinetic energy approaches the Carnot efficiency at $\epsilon = \epsilon_m$, at which no power output occurs. The efficiency η is a peaked function of the barrier height ϵ which is dependant on values of τ, x and μ.

Figure 10 shows plot of w, q_H and η versus τ. From the figure, we can see that w, q_H and η change very slowly at small τ and they decreases drastically near $\tau = \tau_m$.

When the load x is less than the maximum load x_m, the motor works as a heat engine. The power output is a peaked function of the load x and the barrier height ϵ.

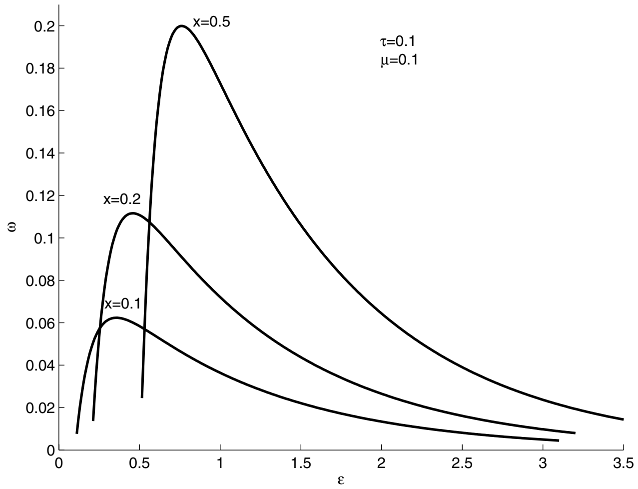


Fig. 8. Dimensionless power output w vs. barrier height ϵ for different values of $x = 0.1$, 0.2, 0.5 ($\tau = 0.1$, $\mu = 0.1$).

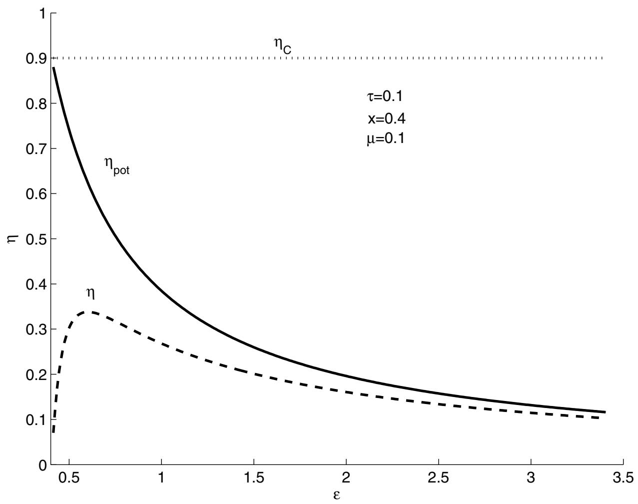


Fig. 9. η_C, η_{pot} and η vs. the barrier height ϵ ($\tau = 0.1$, $x = 0.4$, $\mu = 0.1$).

The efficiency η is less than the efficiency η_{pot} and can never approach the Carnot efficiency η_C. The heat flow via kinetic energy causes the energy leakage unavoidably and reduces the efficiency of the heat engine.

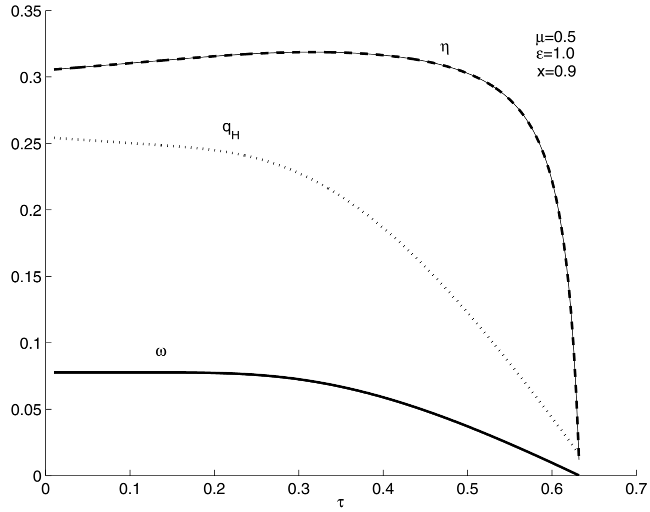


Fig. 10. Dimensionless heat flow q_H,q_C and power output w vs. τ ($x = 0.9$, $\epsilon = 1.0$, $\mu = 0.5$).

3 Concluding remarks

In present work, we study the energestic of a thermal motor which consists of a Brownian particle moving a sawtooth potential with an external load where the viscous medium is alternately in contact with hot and cold reservoirs along the space coordinate. We make a clear distinction between the heat flow via the kinetic and the potential energy of the particle, and show that the former is always irreversible and the latter may be reversible when the engine works quasistatically.

When the external load is not enough, the thermal motor can work as a heat engine. The power output has a maximum value at certain value of the load x which dependant on the others parameters. If only the heat flow via potential is considered, the efficiency η_{pot} increases with the load x and approaches the Carnot efficiency η_C under quasistatic condition, which agrees with the previous work. When the heat flow via kinetic energy is also considered, the efficiency η is less than η_{pot} and can never approach the Carnot efficiency η_C. It is also found that the structure of the potential decides the maximum load capability of the heat engine. The heat flow via potential is reversible, while the heat flow via kinetic energy is irreversible. The heat flow via kinetic energy reduces the heat engine efficiency.

The thermal motor was initially proposed in an attempt of describing molecular motors in biological systems. The heat flow via energy is irreversible, so the efficiency (COP) can not approach the the Carnot efficiency (COP) at a quasistatic condition. However, Molecular motors are known to operator efficiency. They can convert molecular scale chemical energy into macroscopic mechanical work with high efficiency in water at room temperatures, where the effect of thermal fluctuation is unavoidable. Thus, the next challenging question would be "How to reduce the heat flow via kinetic energy?".

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